

Association of Mathematical Communication and Problem Solving Abilities: Implementation of MEAs Strategy in Junior High School

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Abstract

This paper reports the result of study, which examines the association of problem solving and mathematical communication abilities of students after going through a Mathematical Eliciting Activities (MEAs) strategy. This quasi-static research involves 60 students of Junior High School in Depok, West Java, Indonesia. The data was collected through instruments that include the tests of the prior knowledge of mathematics, the problem solving and the mathematical communication. The data was analyzed by Pearson-Chi Square's Test. The results inform that there is a significant association between mathematical problem solving and communication ability of students after going through MEAs strategy. The strength of the association between mathematical problem solving and communication can be seen from the students problem solving abilities and followed by their mathematical communication abilities.

Keywords: Association, Mathematical communication, Model Eliciting Activities, Problem solving

Introduction

The goal of education for students (NCTM, 1989) is that education should motivate student' interest to enhance their mathematical literacy. The two goals cited are to enable students to become mathematical problem solvers and communicate mathematically well. Mathematical problem-solving activity is core of mathematical teaching-learning. Other activity also important is challenging students to think and reason about mathematics and to communicate their mathematical thinking orally and in writing.

Problem solving (NCTM, 1989), as a process that encompasses the entire process of teaching-learning provides skills about a contextual concept (Polya, 1973) which can be studied through an effort to find a way out which is not immediately achieved. Krulik & Rudnick in Yee (2012) also stated that solving the problem as a new attempt to solve problems by using the knowledge, skills and understanding gained previously. Garofalo & Lester (Kirkley, 2003) stated that mathematical problems solving involves higher-level thinking skills such as visualization, association, reasoning, manipulation, abstraction, analysis, synthesis, and generalization.

Mathematical teaching-learning process that involves a problem solving process can develop a habit of thinking and behaving in mathematics students. Students are trained to develop the mathematical ability by reading, hearing, thinking mathematically, solving problems and validating solutions. This habituation helps students develop problem-solving

skills and retain them. Habit of thinking and behaving mathematics allow students to develop and deepen their mathematical knowledge.

Mathematical problem-solving ability is not a skill that can be acquired immediately. It must be developed through training and habituation carried out repeatedly and continuously. Therefore, teachers should consider designing a teaching-learning process that potentially motivates students to be brave and confident about their ability to solve the problem. Student who studies mathematics should engage actively in thinking about mathematical ideas and constructing mathematical ability.

Once the students are given a problem, students will go through the process of solving problem (Polya, 1973) namely (i) understanding the problem, (ii) making a plan and design how to solve problem, (iii) carrying out the plan, and the final step (iv) looking back, whether settlement proceeds in accordance with what is known and asked. If students have not received the correct completion solution, the students need to look back on a given problem, and then solve it back through the four stages in sequence until the settlement has been properly obtained.

The process of mathematical problem solving by students would be more effective if carried out through discussion. The discussion became a forum for students to speak up about mathematical thinking and learn to understand mathematical thinking other friends. In the process of the discussion, students will a richer understanding of mathematics because students have the opportunity to know the thinking and mathematical ability of their peers. The process of sharing knowledge and mathematical ability in discussions develop and strengthen students' mathematical communication abilities.

Mathematical problem solving abilities of students (NCTM , 2000) can be developed through a process of learning by problem-solving activities that motivate students : (i) build new mathematical knowledge through problem solving ; (ii) solving problems that arise in mathematics and in other contexts ; (iii) apply and adapt a variety of appropriate strategies to solve problems ; (iv) monitor and reflect on the process of mathematical problem solving. While the mathematical communication abilities of students (NCTM, 2000) can be developed through a mathematical teaching and learning process that allows all students: (i) to organize and consolidate their mathematical thinking through communication; (ii) to communicate their mathematical thinking coherently and clearly to peers , teachers , and others ; (iii) to analyze and evaluate the mathematical thinking and strategies of others; (iv) to use the language of mathematics to express mathematical ideas precisely.

One of the strategies of teaching-learning process is potential for students to explore mathematical skills in solving mathematical problems and to communicate mathematically is Model Eliciting Activities (MEAs). MEAs has the advantage that the learning process in the classroom, learning outcome, which is shown by student indicated more than just answering questions with short answers and narrow.

The Model Eliciting Activities (MEAs)

Eric (2008), in his study of the implementation of MEAs in the teaching-learning process of mathematics at the elementary school level, revealed that the implementation of MEAs allows students to be involved in mathematical activity to solve contextual problem, which is not provided by the conventional teaching-learning process in conventional. Process of problem

solving in MEAs is (Moore & Dux, 2004) conditioned for students to create a model and to demonstrate their mathematical thinking in writing.

Ekmekci and Krause (2011) suggested that the MEAs motivates students to describe, re-test, and refine their mathematical thinking. Moreover, Ekmekci and Krause also suggested that MEAs makes students use the media representation and record a system concept that is used to be applied by the students in writing. Description about a potential of MEAs suggest that teaching-learning process of MEAs gives chance for student to develop mathematical problem-solving skills.

Lesh, et al. (2000) designed and tested the phases of the MEAs which make students understand about mathematical concepts which are based on the six principles of MEAs. The 1st principle is the construction of models that highlight the problems designed to allow the creation of models related to: the elements, relations and operations between elements, patterns and rules that govern this relationship.

The second principle is the principle of reality which emphasizes the issues that should be meaningful, relevant and based on the real data for students or slightly altered. Solutions must be "real" and relevant in the everyday life of students. Therefore, the context of the situation must be reasonable in terms of knowledge and real-life experience.

The third principle is self-assessment that requires students to be able to assess themselves and to measure usefulness of their solutions. Students should be able to: detect flaws when conceptualizing, comparing the most promising alternative solution, integrating force among alternatives, minimizes a weakness, expand and improve promising alternative solutions, and assess solutions that have been obtained.

The fourth principle is model documentation that requires students to express and document their thought processes in their solutions. The fifth principle is ability to share and re-use model that ensures solutions created by students can be generalized or easily adapted to other similar situations. This principle also ensures that the resulting model can be communicated to other students in ways that are clearly understood and enable their solutions can be used by others.

The sixth principle is an effective prototype ensures that the resulting model will be as simple as possible, but still mathematically significant. Models, which should represent the major ideas, prototypes or metaphor, should provide teaching-learning process useful for interpreting other problems that have the same basic structure.

In MEAs, at the end of the modeling process, students are expected to construct a mathematical model that can be shared and reused. Moreover, the model is very likely, cyclical and repetitive, and students make the extension, revision, repair or rejection of their earlier models (Zbiek & Conner, 2006). This shows that the mathematical model is a non-linear process, including the steps are interrelated.

There are five basic steps in the process of mathematical modeling (NCTM, 1989), namely: (i) identifying and simplify real-world problem situations, (ii) developing a mathematical model, (iii) changing and solve the model, (iv) interpreting the model, and (v) validating the model. Stages of modeling in the picture below are one of the learning activities that will be appeared in the process of a teaching-learning model through MEAs.

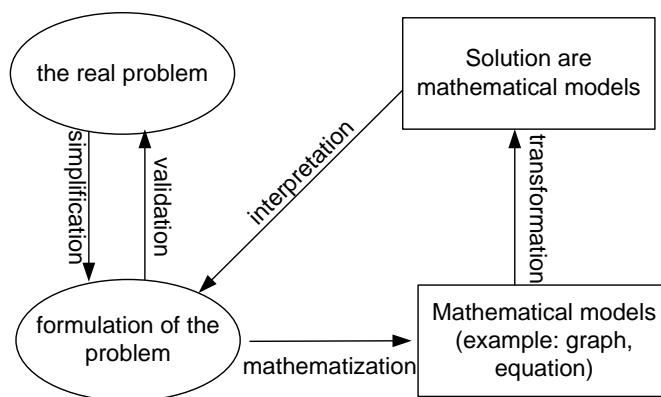


Figure 1. Models of standard process modeling (Zbiek & Conner, 2006).

In the first step, students identify real-world problem situations that must be resolved appropriately. This process also includes a "decisive action" because the students determine the conditions and assumptions relating to the situation in order to consider and use in the next step that is to build a mathematical model (Zbiek & Conner, 2006).

In the second step, the students create a mathematical representation of the components of problems and the relationships between them. In this step, students define variables, notation, and explicitly identify several forms of mathematical relationships and structures, create charts, and write equations. In the description of the modeling process, Zbiek and Conner (2006) describe this process as finding mathematical properties and parameters related to the conditions and assumptions that have been identified previously.

In the transformation step, students analyze and manipulate mathematical models to find significant solutions from problems that have been identified. This step is usually familiar to students. Models of the second step completed, and the answer is understood in the context of the original problem.

In the interpretation step (Hodgson, 1999 in Zbiek and Conner, 2006), students bring back into the solutions they obtain into context of mathematical models related to real-world problem situations that have been formulated. Then, they test and evaluate whether the obtained solution is meaningful related to the problem. This step is similar to the process of mathematical modeling; students are challenged to develop a relationship between mathematical model and real world (Zbiek & Conner, 2006).

In the last step (Hodgson, 1999 in Zbiek and Conner, 2006), students also think about the validity and usefulness of the created model. Lesh and Doerr (2003) described the process of "verification" requires students to test predictions and conclusions obtained through validity into the real-world situation. Model is evaluated about its consistency of the specific objectives that have been determined (Zbiek & Conner, 2006). This procedure makes the model be considered as a powerful model (Lesh & Doerr, 2003).

Associated with research on the use of a teaching-learning process through MEAs, there are some studies that can be used as comparisons. Eric (2008) examined the use of MEAs in the teaching-learning process of mathematics in primary schools. Mathematics teaching-learning process considered takes place contextually by modeling activity as a catalyst to bring mathematical reasoning and make the lessons meaningful. MEAs gives students the

opportunity to resolve the really contextual issues. In addition students gain the opportunity to develop mathematical thinking in the modeling process.

Research done by Yildirim et al. (2010) obtains findings related to the benefits of the implementation of MEAs, namely the implementation of MEAs would be beneficial to the fullest whether it includes coaching by the instructor in the learning process. The results also strongly recommend the use of MEAs to help teachers evaluate the students' problem-solving.

Discussion in the MEAs strategy, which is influenced by the six principles of the MEAs, is believed to have the potential to develop the mathematical communication and problem solving ability simultaneously. Mathematical communication and problem solving ability in discussion will appear simultaneously. This happens because when students communicate mathematics to solve problems with other friends, their mathematical communication ability becomes better. The association between mathematical communication and problem solving ability in this process can be described as two gear machinery ([Icon-gears2.svg](http://commons.wikimedia.org/wiki/File:Icon-gears2.svg), source from: <http://commons.wikimedia.org/wiki/File:Icon-gears2.svg>) each other drives mutually.

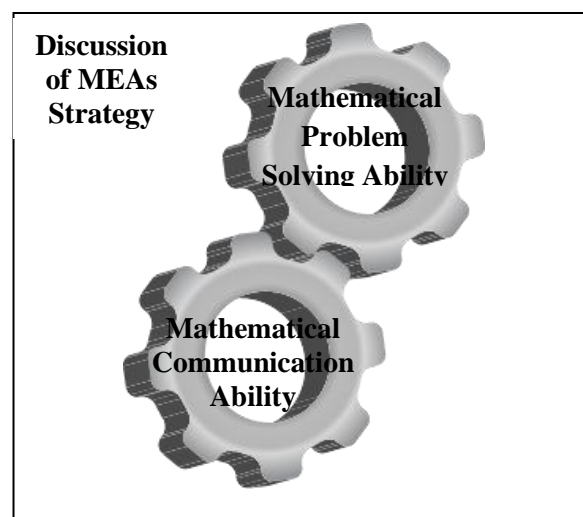


Figure 3. The mutually driving of problem solving and communication mathematics

Assume that the MEAs has the potential to develop problem-solving and mathematical communication skills, motivates researcher conduct a study entitled "Association of Mathematics Problem Solving and Communication: The Implementation of MEAs Strategy", in order to determine whether there is an association between the ability of mathematical problem solving and students' communication after going through MEAs strategy.

Methodology

This research is a quasi-experiment. This research used two schools as a research subject groups based on the recommendations of Depok city Education Department, West Java. Both schools were used as research subjects are divided into two categories. One school has high category, while the other school has medium category based on categorization determined by the education office of Depok city government.

Research implementation in both schools is done directly by the researcher, so that the determination of the class of research customized with setting timetable in order not clash each

other between timetables in the class of each schools. Before the treatment, all students are tested for their prior knowledge of mathematics.

The test of students' mathematical prior knowledge is grouped into three categories i.e. high, medium, and low as in the Table 1.

Table 1. *Criteria of Category of the Students' Mathematical Prior Knowledge*

Communication Abilities	Problem Solving Abilities			
	High	Middle	Low	Total
High	5	2	1	8
Middle	7	3	13	23
Low	2	4	23	29
Total	14	9	37	60

The teaching-learning process used two materials. Those are: Implementation Plan for teaching-learning process (RPP) and the Student Worksheet (LKS). Plan of the implementation teaching-learning process (RPP) and the student worksheet (LKS) is designed to train students to develop their skills to solve a mathematical problem about linear equations and communicate their mathematical thinking into writing.

Students are actively involved in teaching-learning process to make mathematical models of a given problem using learning resources (LKS). As a source of student learning in this study, LKS contains contextual problem that come with the questions that lead students to their ability in developing mathematical models for solving the problem. After the teaching-learning process is over, both groups were given the same test for examining the problem solving abilities and the mathematic communication abilities in writing.

According to Sumarmo (2008) indicator of the ability of solving mathematical problems include: (i) the student can identify the elements that are known, (ii) the student can formulate a mathematical problem or develop a mathematical model, (iii) the student can apply strategies to solve a variety of problems (similar and new problems) within or outside of mathematics, (iv) the student can explain and interpret the results as the origin of the problem, and (v) the students can use math significantly. Scoring to mathematical problem-solving ability given in a scale of 10 which is a modification of the Scale for Problem Solving (Szetela etc., 1992) in Mathematical Problem Solving Rubric Scale Chicago as in Table 2. below. The numbers of questions used to test the ability of mathematical problem solving are five questions.

Table 2. *Item-Scoring Guidelines Test of Mathematical Problem Solving Ability*

Score	Scale I	Scale II	Scale III
	Understand	Plan	Complete
0	There is no attempt	There is no attempt	There is no attempt
1	Completely wrong to interpret the problem	Solution plan does not fit	Computational errors, most of the solution is wrong, wrong answer.
2	Misinterpretation of most of the problems	Partially correct procedures with large error	Completion true, true answer
3	Misinterpretation of fraction problems	Substantially correct procedure with minor errors	
4	Complete understanding of the problem	Solution plan properly without arithmetic errors	
Max score	4	4	2

The indicator shows the mathematical communication skills (Sumarmo, 2008) are: (i) link real objects, drawings, and diagrams into mathematical ideas, (ii) explain the ideas, situations and mathematical relationships, orally or in writing with real objects, images, and graphs; (iii) declare everyday problems with mathematical language or symbols, (iv) listen, discuss, and write about mathematics; (v) read with understanding a written mathematical presentation; (vi) make conjectures, formulate arguments, formulate definitions, and generalizations, and (vii) describe or paraphrase a paragraph in the language of mathematics itself. However, this study only tested the students' mathematical communication ability in writing. Score of mathematical communication ability is given in a scale of zero to four which is a modification of Maryland State Department of Education (1991): "Sample activities, student responses and Maryland teachers' comments on a sample task: Mathematics Grade 8". The lowest possible score achieved is zero, and the highest score that can be achieved by students is four. The number of questions used to test the ability of mathematical communication is five questions.

Table 3. *Item-Scoring Guidelines Test of Mathematical Communication Ability*

Score	Mathematical Communication Ability in Writing
0	Empty, or the answer is not enough to get a score
1	The answer is not correct, the effort made is not true
2	The use of mathematical language (terms, symbols, signs and / or representations) are minimally effective and accurate, to describe operations, concepts, and process
3	The use of mathematical language (terms, symbols, signs, and / or representation) is most effective, accurate, and thorough to describe operations, concepts and processes.
4	The use of mathematical language (terms, symbols, signs, and / or representation) is very effective, accurate, and thorough, to describe operations, concepts, and processes.

The Result of Study

Statistics of the students' mathematical prior knowledge (PAM) students described in Table 4. below.

Table 4. *Statistics of PAM Scores by School Level*

Statistics	Level of School	
	High	Medium
The number of students	34	26
Maximum Score	20	18
Minimum Score	6	6
Average Score	13.79	11.65
Standard Deviation	2.76	3.72

The students' PAM at the high-level school have an average of mathematical prior knowledge higher than the students' PAM at the medium-level school (13.76 : 11.65), and the standard deviation of the students' PAM at the high-level school is lower than the standard deviation of the students' PAM students at the medium-level school (2.76 : 3.72). The standard deviation obtained explains that the students' PAM at the high-level school more homogeneous than the students' PAM at the medium-level school.

Statistics used to describe the students' mathematical problem-solving ability presented in Table 5. This table contains data about the number of students, the average score, and standard deviation of the test results of mathematical problem-solving abilities.

Table 5. *Statistics of the Mathematical Problem-Solving Ability*

Statistics	PAM of Students at Upper-Level School				PAM of Students at Medium-Level School			
	High	Middle	Low	Total	High	Middle	Low	Total
The Number of Students	13	16	5	34	6	10	10	26
Average	37.54	25.87	25.83	30.32	27.67	23.5	20.3	23.23
Standard Deviation	7.05	7.88	8.98	9.45	12.75	8.86	10.13	10.29
The ideal score is 50								

Statistics of the mathematical communication ability of students are described in Table 6. The highest total score that can be achieved by students for mathematical communication ability (the ideal score) is 20.

Table 6. *Statistics of the Mathematical Communication Ability*

Statistics	PAM of Students at Upper-Level School				PAM of Students at Medium-Level School			
	High	Middle	Low	Total	High	Middle	Low	Total
The Number of Students	13	16	5	34	6	10	10	26
Average	15.15	10.67	10.67	12.38	9.83	6.20	5.60	6.81
Standard Deviation	2.79	3.72	3.56	3.98	6.85	2.90	2.76	4.25
The ideal score is 20								

The scores of mathematical problem solving ability and communication mathematical learning of students who got the MEAs at both school levels were converted into categories and presented into a contingency table. The data in the contingency table was tested to determine whether the two type of scores are mutually revealing students' abilities association or not. Grouping of categorical data are summarized in the following contingency table (Table 7.).

Table 7. *Contingency Table of the Mathematical Problem Solving and Communication Ability*

Problem Solving Abilities Communication Abilities	High	Middle	Low	Total
High	5	2	1	8
Middle	7	3	13	23
Low	2	4	23	29
Total	14	9	37	60

Pearson-Chi Square's test is used to test whether there is an association between the mathematical problem solving ability and mathematical communication ability gives the results as shown in Table 8.

Table 8. *Associations between Mathematical Problem Solving and Communication Ability*

Pearson-Chi Square's Test		
χ^2	df	Asymp.Sig (2-sided)
14.433	4	0.006

Values presented in Table 8. $\chi^2 = 14.433$ with Asymp.Sig (2-sided) which is smaller than $\alpha = 0.05$. This alpha value causes the rejection of H_0 . The test results reject H_0 gives a conclusion about the existence of association between the ability of solving mathematical problems and students' mathematical communication at both school levels is significant.

The strength or weakness of the size of the known association contingency coefficient calculation results of SPSS as shown in Table 9.

Table 9. *Coefficient of Contingency for Association between Mathematical Problem Solving and Communication ability*

	Value	Approx. Sig.
Contingency Coefficient (C)	0.440	0.006
N of Valid Cases	60	

Based on the calculation of the coefficient of contingency research data as shown in Table 8, i.e. $C = 0.440$ with P-value = 0.006, explains the presence of a positive significant association between the ability of mathematical problems solving and communication. These results explain that the MEAs strategy has the potential to develop communication mathematics abilities of students who tend to be followed also by their problem solving abilities. The problem solving abilities of students also tend to be followed by their mathematical communication abilities as well. The following illustration is the result of the work of two students in solving the money problem.

Money problem. Bayu and Doni each have some money. If Bayu gave Rp 3,000 to Doni, the Doni money is to be twice as much money as Bayu has (after Bayu gave his money Rp 3,000 to Doni). However, when Bayu receives Rp 1,000 from Doni, Bayu's money will be 3 times as much money as Doni has (after Doni gave his money Rp 1,000 to Bayu). How much money is owned by Bayu and Doni?

Handwritten mathematical solution for the money problem by Student 1. The student defines Bayu's money as x and Doni's money as y . They set up two equations based on the problem conditions:

$$y + 3000 = 2(x - 3000)$$

$$x + 1000 = 3(y - 1000)$$

The student then simplifies these equations and uses the elimination method to solve for x and y .

$$\begin{aligned} y + 3000 &= 2(x - 3000) \\ y + 3000 &= 2x - 6000 \\ y - 2x &= -6000 - 3000 \\ y - 2x &= -9000 \\ \Rightarrow -2x + y &= -9000 \end{aligned}$$

$$\begin{aligned} x + 1000 &= 3(y - 1000) \\ x + 1000 &= 3y - 3000 \\ x - 3y &= -3000 - 1000 \\ x - 3y &= -4000 \end{aligned}$$

The student then performs the elimination method:

$$\begin{array}{r} -2x + y = -9000 \quad | \times 3 | \quad -6x + 3y = -27000 \\ x - 3y = -4000 \quad | \times 1 | \quad x - 3y = -4000 \\ \hline -5x = -31000 \\ x = \frac{31000}{5} = 6200 \end{array}$$

Substituting $x = 6200$ into the second equation:

$$\begin{aligned} x - 3y &= -4000 \\ 6200 - 3y &= -4000 \\ -3y &= -4000 - 6200 \\ -3y &= -10200 \\ y &= 3400 \end{aligned}$$

The final answer is: Bayu = 6200, Doni = 3400.

Figure 3. The Answer of Student 1

Handwritten mathematical solution for the money problem by Student 2. The student defines Bayu's money as x and Doni's money as y . They set up two equations based on the problem conditions:

$$y + 3000 = 2(x - 3000)$$

$$x + 1000 = 3(y - 1000)$$

The student then simplifies these equations and uses the elimination method to solve for x and y .

$$\begin{aligned} y + 3000 &= 2(x - 3000) \\ y + 3000 &= 2x - 6000 \\ y - 2x &= -6000 - 3000 \\ y - 2x &= -9000 \\ \Rightarrow -2x + y &= -9000 \end{aligned}$$

$$\begin{aligned} x + 1000 &= 3(y - 1000) \\ x + 1000 &= 3y - 3000 \\ x - 3y &= -3000 - 1000 \\ x - 3y &= -4000 \end{aligned}$$

The student then performs the elimination method:

$$\begin{array}{r} -2x + y = -9000 \quad | \times 3 | \quad -6x + 3y = -27000 \\ x - 3y = -4000 \quad | \times 1 | \quad x - 3y = -4000 \\ \hline -5x = -31000 \\ x = \frac{31000}{5} = 6200 \end{array}$$

Substituting $x = 6200$ into the second equation:

$$\begin{aligned} x - 3y &= -4000 \\ 6200 - 3y &= -4000 \\ -3y &= -4000 - 6200 \\ -3y &= -10200 \\ y &= 3400 \end{aligned}$$

The final answer is: Bayu = 6200, Doni = 3400.

Figure 4. The Answer of Student 2

The mathematical problem solving ability of students on issues SPLDV in Figure 3 and 4 shows the performance of the students. It appears that they understand the problem. Understanding the problem is the ability that should be mastered early. At the next step, students are able to plan the completion of the modeling information in the form of mathematical models to the problems correctly. Dispute resolution as the final step, can be

performed by students in accordance with the correct mathematical procedures correctly by using mathematical concepts such as rules related commutative and distributive. Their ability to solve the problem of generating a number that can solve the problem. The first student looks more communicative in elaborate mathematical thinking, while the second student answer mathematical problems without connecting solution toward the problem context.

Discussion

Modeling process engages students in an experience of a mathematical modeling process actively that makes students are trained to develop a model. Students should explore and map their mathematical knowledge and apply it to interpret, analyze, explain, make conjecture, compare, and give a decision on their mathematical thinking in solving a given problem.

Each group consists of students with prior mathematical knowledge. Teacher as facilitator in the learning process bridges the gap among students, so that all students have an understanding. During the study, it appears that grouping students effectively motivate students to actively engage in learning activities.

To keep students' involvement during the teaching-learning process is not disturbed; teachers' intervention to motivate students is needed in order students work as designed. Teachers monitor mathematical activities of working groups, gave feedback as reinforcement, ask questions or give some examples and non-examples. Intervention is used to provoke and broaden student' understanding of mathematics by providing arguments, asking questions and paying attention to mathematical thinking of other students who lead to find concepts, models and solutions.

MEAs strategy is potential to construct abilities of problem solving and communication mathematics. Modeling and testing of the model is the focus of MEAs teaching and learning activities (the MEAs principle of mathematical modeling). Mathematical models use to define the relationship between the elements, define the operation about how the elements interact in the problem, and identify patterns or rules applicable to the relationship and operations. Hence, the final product of the MEA is a mathematical model (Lesh et al., 2000). Teachers apply the model construction principles of MEAs as orientation in guiding the students during the learning process. Modeling (Blomh 2004, in Eric 2008) gives students experience to understand and describe the relationship between mathematical and their daily lives (the MEAs principle of reality), so that their motivation to learn mathematics increased.

Conclusion

The results of this study indicate that the learning of mathematics with MEAs strategy has the potential to develop mathematical problem-solving ability and mathematical communication simultaneously. This is shown by the results of the association test of the ability of problem solving and mathematical communication. Association test results explaining the existence of a positive association between students' mathematical problem solving skills and communication.

Implications

The study implies the development of teaching materials for MEAs teaching-learning process should pay attention to the differences of students' ability. Steps of the teaching-learning process translated into mathematical worksheets (LKS) help students to build knowledge. In addition to structured worksheets, students who are weak in mathematics are given the opportunity to work in a group with students who have strong mathematical ability in order to get a chance to think and argue with peers. Students will be interested in mathematics and their

mathematical ability grows. Discussion groups provide an opportunity for students who have high mathematical ability to strengthen their understanding through their belief in helping to explain mathematical concepts to other students in need. Meanwhile, students who have a weak mathematical ability will have an opportunity to develop knowledge and mathematical ability through discussions with their smarter peers.

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